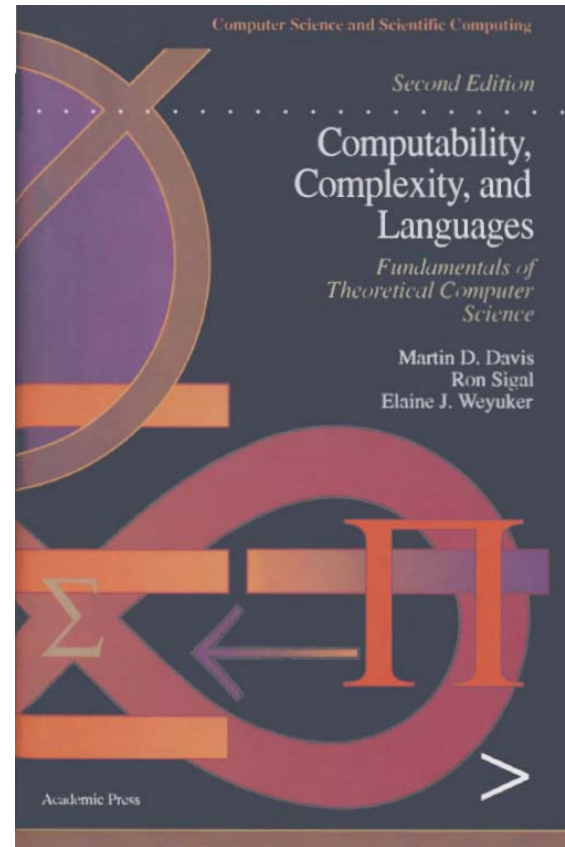
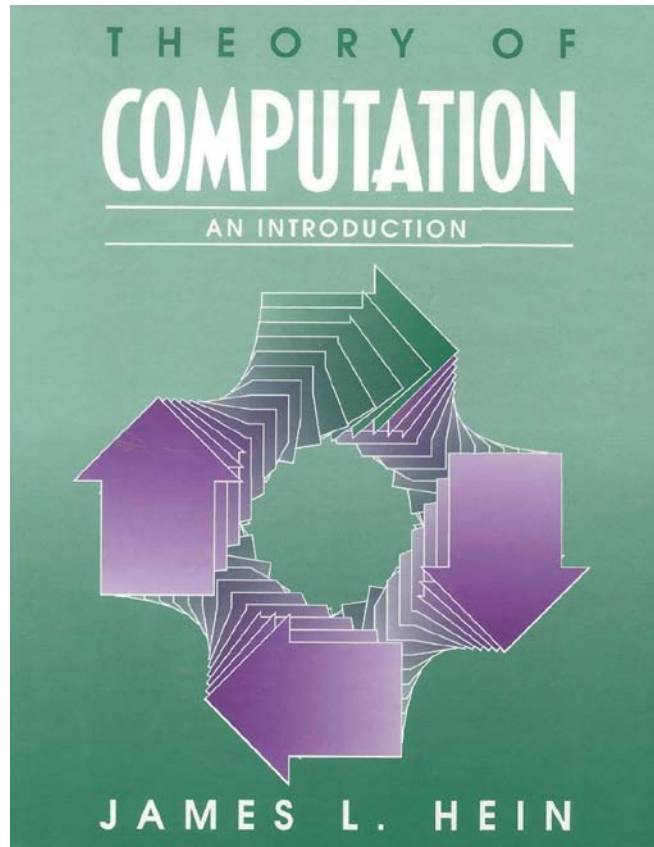


Theory of Computation

Lecture 05

Books



PowerPoint

<http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767>

The screenshot displays a web interface for Benha University. At the top, the university logo and name are on the left, and a blue navigation bar contains the text 'Staff Search: Welcome: Ahmed Hassan Ahmed Abu El Atta (Log out)'. Below the navigation bar, a sidebar on the left lists various user options such as 'Home', 'My C.V.', 'About', 'Courses', 'Publications', 'Inlinks(Competition)', 'Theses', 'Reports', 'Published books', 'Workshops / Conferences', 'Supervised PhD', 'Supervised MSc', 'Supervised Projects', 'Education', 'Language skills', 'Academic Positions', and 'Administrative Positions'. The main content area shows the user's current location: 'You are in: Home/Courses/Automata and Formal Languages', with a 'Back To Courses' link. Below this, the course title 'Ass. Lect. Ahmed Hassan Ahmed Abu El Atta :: Course Details: Automata And Formal Languages' is displayed. A table provides course information: 'Course name' (Automata and Formal Languages), 'Level' (Undergraduate), 'Last year taught' (2018), and 'Course description' (Not Uploaded). A 'Course password' field is present but empty. Below the table, there are sections for 'Course files', 'Course URLs', 'Course assignments', and 'Course Exams & Model Answers', each with an 'add' link. On the right side of the page, there are social media icons for Google, a book icon, RG, LinkedIn, Facebook, Twitter, Google+, YouTube, WordPress, Instagram, a soccer ball icon, a question mark icon, and a speech bubble icon labeled '(edit)'. The page is styled with a light blue and white color scheme.

Benha University

Staff Search: Welcome: Ahmed Hassan Ahmed Abu El Atta (Log out)

You are in: [Home/Courses/Automata and Formal Languages](#) [Back To Courses](#)

Ass. Lect. Ahmed Hassan Ahmed Abu El Atta :: Course Details:
Automata And Formal Languages

[add course](#) | [edit course](#)

Course name	Automata and Formal Languages
Level	Undergraduate
Last year taught	2018
Course description	Not Uploaded

Course password

Course files	add files
Course URLs	add URLs
Course assignments	add assignments
Course Exams & Model Answers	add exams

(edit)

Programs and Computable Functions

SIMPLE LANGUAGE III

Agenda

- Pairing Functions
- Examples
- Gödel Number
- Examples
- Coding Programs by Numbers
- Examples

Pairing Functions

The Pairing Functions is for coding pairs of numbers by single numbers.

$$\langle x, y \rangle = 2^x(2y + 1) - 1$$

If z is any given number, there is a unique solution x, y to the equation

$$\langle x, y \rangle = z$$

x is the largest number such that $2^x \mid (z + 1)$, and y is then the solution of the equation

$$2y + 1 = (z + 1)/2^x$$

Example $\langle x, y \rangle = ??$

$\langle 2, 3 \rangle$

=

$\langle 3, 2 \rangle$

=

$\langle 0, 5 \rangle$

=

$\langle 4, 0 \rangle$

=

Example $\langle x, y \rangle = ??$

$$\langle 2, 3 \rangle$$

$$= 2^2(2*3+1)-1$$

$$= 4(7)-1 = 27$$

$$\langle 0, 5 \rangle$$

$$= 2^0(2*5+1)-1$$

$$= 1(11)-1 = 10$$

$$\langle 3, 2 \rangle$$

$$= 2^3(2*2+1)-1$$

$$= 8(5)-1 = 39$$

$$\langle 4, 0 \rangle$$

$$= 2^4(2*0+1)-1$$

$$= 16(1)-1 = 15$$

Example $z = ??$

5

$$= 2^x(2y + 1) - 1$$

100

$$= 2^x(2y + 1) - 1$$

Example $z = ??$

5

$$= 2^x(2y + 1) - 1$$

6

$$= 2^x(2y + 1)$$

$$2^{1*3} = 2^x(2y + 1)$$

$$x = 1,$$

$$2y+1 = 3$$

$$y = 1$$

$$5 = \langle 1, 1 \rangle$$

100

$$= 2^x(2y + 1) - 1$$

101

$$= 2^x(2y + 1)$$

$$2^{0*101} = 2^x(2y + 1)$$

$$x = 0,$$

$$2y+1 = 101$$

$$y = 50$$

$$100 = \langle 0, 50 \rangle$$

Example $z = ??$

11

$$= 2^x(2y + 1) - 1$$

22

$$= 2^x(2y + 1) - 1$$

Example $z = ??$

11

$$= 2^x(2y + 1) - 1$$

$$12 = 2^x(2y + 1)$$

$$2^{2*3} = 2^x(2y + 1)$$

$$x = 2,$$

$$2y+1 = 3$$

$$y = 1$$

$$11 = \langle 2, 1 \rangle$$

22

$$= 2^x(2y + 1) - 1$$

$$23 = 2^x(2y + 1)$$

$$2^{0*23} = 2^x(2y + 1)$$

$$x = 0,$$

$$2y+1 = 23$$

$$y = 11$$

$$22 = \langle 0, 11 \rangle$$

Gödel Number

We define the Gödel number of the sequence (a_1, \dots, a_n) to be the number

$$[a_1, \dots, a_n] = \prod_{i=1}^n p_i^{a_i}$$

Example $[a, b, c, \dots] = ??$

$[2, 3] =$

$[0, 2, 3] =$

$[2, 3, 0, 0, 0] =$

Example $[a, b, c, \dots] = ??$

$$\begin{aligned}[2, 3] &= 2^2 * 3^3 \\ &= 4 * 27 = 108\end{aligned}$$

$$\begin{aligned}[0, 2, 3] &= 2^0 * 3^2 * 5^3 \\ &= 1 * 9 * 125 = 1125\end{aligned}$$

$$\begin{aligned}[2, 3, 0, 0, 0] &= 2^2 * 3^3 * 5^0 * 7^0 * 11^0 \\ &= 4 * 27 * 1 * 1 * 1 = 108\end{aligned}$$

Example [??, ??, ??] = P

30 = []

120 = []

Example [??, ??, ??] = P

$$30 = 2^1 * 3^1 * 5^1$$
$$= [1, 1, 1]$$

$$120 = 2^3 * 3^1 * 5^1$$
$$= [3, 1, 1]$$

Example [??, ??, ??] = P

39 =

Example [??, ??, ??] = P

$$39 = 3 * 13$$

$$= 2^0 * 3^1 * 5^0 * 7^0 * 11^0 * 13^1$$

$$= [0, 1, 0, 0, 0, 1]$$

Coding Programs by Numbers

We are going to associate with each program \mathcal{P} of the language \mathcal{S} a number, which we write $\#(\mathcal{P})$, in such a way that the program can be retrieved from its number. To begin with we arrange the variables in order as follows:

$$Y \ X_1 \ Z_1 \ X_2 \ Z_2 \ X_3 \ Z_3 \ \dots$$

Coding Programs by Numbers

Next we do the same for the labels:

$$A_1 \ B_1 \ C_1 \ D_1 \ E_1 \ A_2 \ B_2 \ C_2 \ D_2 \ E_2 \ A_3 \dots$$

We write $\#(V)$, $\#(L)$ for the position of a given variable or label in the appropriate ordering. Thus $\#(X_2) = 4$, $\#(Z_1) = \#(Z) = 3$, $\#(E) = 5$, $\#(B_2) = 7$.

Coding Programs by Numbers

Now let I be an instruction (labeled or unlabeled) of the language \mathcal{S} . Then we write

$$\#(I) = \langle a, \langle b, c \rangle \rangle$$

where

1. if I is unlabeled, then $a = 0$; if I is labeled L , then $a = \#(L)$;
2. if the variable V is mentioned in I , then $c = \#(V) - 1$;
3. if the statement in I is

$$V \leftarrow V \quad \text{or} \quad V \leftarrow V + 1 \quad \text{or} \quad V \leftarrow V - 1,$$

then $b = 0$ or 1 or 2 , respectively;

4. if the statement in I is

IF $V \neq 0$ GOTO L'

then $b = \#(L') + 2$.

Coding Programs by Numbers

Finally, let a program \mathcal{P} consist of the instructions I_1, I_2, \dots, I_k . Then we set

$$\#(\mathcal{P}) = [\#(I_1), \#(I_2), \dots, \#(I_k)] - 1$$

Example

$$X \leftarrow X + 1$$

$$[A] \quad X \leftarrow X + 1$$

Example

$$X \leftarrow X + 1$$

$$[A] \quad X \leftarrow X + 1$$

$$\langle 0, \langle 1, 1 \rangle \rangle = \langle 0, 5 \rangle = 10$$

$$\langle 1, \langle 1, 1 \rangle \rangle = \langle 1, 5 \rangle = 21$$

[A] $X \leftarrow X + 1$
IF $X \neq 0$ GOTO A

[A] $X \leftarrow X + 1$

IF $X \neq 0$ GOTO A

Determine the program
whose number is 199

Determine the program whose number is 199

$$199 + 1 = 200 = 2^3 \cdot 3^0 \cdot 5^2 = [3, 0, 2].$$

Thus, if $\#(\mathcal{P}) = 199$, \mathcal{P} consists of 3 instructions, the second of which is the unlabeled statement $Y \leftarrow Y$. We have

$$3 = \langle 2, 0 \rangle = \langle 2, \langle 0, 0 \rangle \rangle$$

and

$$2 = \langle 0, 1 \rangle = \langle 0, \langle 1, 0 \rangle \rangle.$$

Thus, the program is

$$\begin{array}{l} [B] Y \leftarrow Y \\ \quad Y \leftarrow Y \\ \quad Y \leftarrow Y + 1 \end{array}$$

